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Thermal and mechanical evolution of shear zones

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Abstract—Physical models of geological shear zones are computed taking into account heating by deformation and consequent softening of the rock. The models show that initiation of a ductile shear zone proceeds by the rapid build up of a thermal peak and by concentration of the strain. After this the temperature levels off as the widths of the thermal anomaly and of the sheared volume slowly increase. For imposed velocities compatible with plate motion, shear heating softens the rock efficiently but does not produce melting. This is not true in stratified structures where a sheared hard layer can heat up sufficiently to melt a neighbouring layer. Other situations where melting might occur are also analysed.

INTRODUCTION

THE CONCENTRATION of shear in zones of finite width inside a homogeneous rock mass implies efficient local softening. In particular, deep lithospheric shear zones with displacement velocities of more than 1 cm a^{-1} require a ductility much higher than that of the average lithosphere. Various softening processes can be invoked (Poirier 1980). Here shear heating will be considered. Indeed the deformation produces a considerable amount of heat if stresses of several tens of MPa are involved. The elevation of temperature softens the rock and thus shear tends to concentrate in the hot and therefore more ductile region. Other studies of thermomechanical coupling (Brun & Cobbold 1980) emphasize the difference in behaviour for two types of boundary conditions. If the applied stress is assumed to be constant, thermal runaway may occur. On the other hand, shear heating produces only moderate temperature rises when the chosen boundary condition is a constant velocity (Turcotte & Oxburgh 1968). This type of problem was studied in detail by Yuen *et al.* (1978) for different types of rock. For mathematical convenience this last paper used a linear approximation for the creep law (Newtonian fluid) and presented velocity, temperature and stress profiles across a shear zone which are representative of long term behaviour, but failed to yield a physical picture of the initiation stage of a fault zone. Qualitatively the striking features of these models are the following.

(a) A maximum temperature is achieved in the centre of the shear zone. Its magnitude does not vary with time but depends strongly upon rock type. In fact for velocities corresponding to plate motion, the thermal adjustment leads to a viscosity minimum of the order of 10^{21} poises (10^{20} Pa s), independent of rock type.

(b) Thermal conduction generates a widening of the hot region and hence of the shear zone, proportionally to $\sqrt{\text{time}}$. The shear stress value decreases accordingly.

(c) For a homogeneous rock mass, adequate softening is produced by shear heating for temperatures well below the solidus. Melting by this process thus appears to be an unlikely phenomenon.

A comparison of the models with the geological and geophysical data suffers from the scarcity of quantitative observations. Interpretation of grain sizes in term of stress magnitudes has supported the prediction of a constant stress of several hundreds of bars (several tens of MPa) across the Moine shear zone (Cooper *et al.* 1978). In the Alpine zone of New Zealand the broad metamorphic belt compares well with the models (Scholz & Hanks 1978). The common occurrence of partial melting and of magmas is however in contradiction with statement (c) above. This has prompted us to reappraise some aspects of the problem. In particular the initiation stage has been studied, and non-Newtonian laws of deformation as well as more general boundary conditions have been included.

In the first part of this paper we analyze the starting of a shear zone from a pre-existing mechanical weakness or from a localized temperature variation. The differences in long term behaviour between the Newtonian and non-Newtonian cases are clarified. In the second part of the paper possible melting phenomena are investigated following various lines of reasoning, involving either very narrow shear zones or viscoelastic energy storage leading to transient creep. Finally the shear deformation of a stratified geological structure is shown to give rise to the production of magma on a reasonable scale.

INITIATION AND EVOLUTION OF A BROAD SHEAR ZONE

Large scale shear zones involving the whole lithosphere are characteristic of certain plate boundaries but are also generated by collision within continental plates. Our concern relates to the deep ductile portion of fault zones rather than to the shallower brittle crustal layer. Our models explain the very existence of these structures by the softening of the rock by shear heating leading to strain concentration. Temperature is not the only parameter influencing the plastic deformation of rocks. Pressure effects can be neglected as the depth range is limited. It would be interesting to include strain hardening or strain weakening effects in the models as they may well affect the evolution of shear zones. Material science studies have unfortunately not yet given reliable indications concerning these phenomena, which our models will therefore ignore.

The physical model

Steady state creep experiments give the following relationship between strain rate $\frac{\partial u}{\partial y}$, stress τ_{xy} and temperature T:

$$\frac{\partial u}{\partial y} = \frac{2B}{T} \tau_{xy}^{n} \exp\left(-\frac{Q}{RT}\right) \tag{1}$$

where y is the distance away from the centre of the shear zone defined by the (x, z) plane, u is the velocity in the x direction relative to the middle plane, and R is the gas constant. The constant B, the activation energy Q, and the exponent n are determined experimentally. Here we limit our discussion to wet quartzite and use the values $B = 4.7\ 10^{-17}\ \mathrm{Ks^{-1}dyne^{-2.6}cm^{5.2}}$ ($18.8\ 10^{-15}\ \mathrm{Ks^{-2}\ Pa^{-2.6}}$), $n = 2.6$ and $Q = 55$ kcal/mole ($2.3 \times 10^5\ \mathrm{J\ m^{-1}}$) (Parrish *et al.* 1976). This particular choice does not restrict the qualitative value of our conclusions. Physical quantities are assumed to vary only in the y direction. Thus the temperature equation takes the form:

$$\rho C_p \frac{\partial T}{\partial t} = K \frac{\partial^2 T}{\partial y^2} + \tau_{xy} \frac{\partial u}{\partial y} \tag{2}$$

and the momentum equation is simply

$$\frac{\partial \tau_{xy}}{\partial y} = 0 \tag{3}$$

where ρ is the density taken equal to $3\ \mathrm{g\ cm^{-3}}$ ($3 \times 10^3\ \mathrm{kg\ m^{-3}}$), C_p the thermal capacity of 0.27 cal/g (1120 J $\mathrm{kg^{-1}}$), and K the thermal conductivity (0.006 $\mathrm{cal/cm^{-1}s^{-1}\ k^{-1}}$ or $2.5\ \mathrm{J\ m^{-1}s^{-1}\ k^{-1}}$). At time $t = 0$ an initial temperature profile is chosen and a fixed velocity difference u_0 is imposed between the two sides of the shear zone. Equation (3) implies that τ_{xy} does not vary in space. Thus equation (1) after integration gives: $\frac{u_0}{2} = \tau_{xy}{}^n \int_0^\infty \frac{2B}{T} \exp\left(-\frac{Q}{RT}\right) dy$. This relationship yields the value of the stress once the temperature profile has been computed. Introducing it in equation (2) we find:

$$\rho C_p \frac{\partial T}{\partial t} = K \frac{\partial^2 T}{\partial y^2} + \left[\frac{u_0}{2\int_0^\infty \frac{2B}{T} \exp\left(-\frac{Q}{RT}\right) dy}\right]^{\left(\frac{n+1}{n}\right)} \frac{2B}{T} \exp\left(-\frac{Q}{RT}\right). \tag{4}$$

This expression is numerically integrated in order to get the temperature profile $T(y)$. As we just appreciated, this then yields the stress value. The integration of (1) follows immediately, giving the velocity profile $u(y)$.

Zone of weakness of finite width

First let us examine the evolution of a one-dimensional shear zone where the mechanical deformation is restricted to a width of 10 km. Outside this, the material is rigid. The initial temperature is uniform and equals 650 K. The velocity difference u_0 is $10\ \mathrm{cm\ a^{-1}}$. Figure 1 pictures the temperature and velocity profiles for various times. In the first 0.2 Ma the temperature rises rapidly, the effective viscosity in the centre of the zone drops by more than one order of magnitude and the stress falls well below 1 kbar (100 MPa). The effective viscosity μ is simply the ratio of stress and strain rate. From equation (1) it follows that:

$$\mu = \frac{T}{2B} \frac{1}{\tau_{xy}^{(n-1)}} \exp\left(\frac{Q}{RT}\right). \tag{5}$$

This quantity is thus stress dependent in the non-Newtonian cases ($n \neq 1$). In the initiation stage, shearing first occurs across the whole zone of weakness but gradually concentrates in a narrower domain. In contrast to this, for times approaching 1 Ma and beyond, the sheared region broadens until it reattains the full width of the weakness zone. This is because the temperature now increases much more slowly, but the anomaly flattens. The stress and viscosity values also tend to level off. One may wonder by how much the solutions differ if the initial temperature, the width of weakness, or the velocity are changed within moderate limits. Our computations show that the colder the initial temperature or the narrower the zone of weakness, the higher the initial shear stress will be, and thus the shorter the initiating period and the stronger the concentration of shear. The long term behaviour however is practically independent of imposed initial conditions. Changes in the velocity u_0 turn out to have the long term influence predicted by Yuen *et al.* (1978): smaller velocities lead to colder temperatures and higher viscosities.

Shear concentration by a weak thermal anomaly

The material is now considered as mechanically homogeneous and a velocity $\pm 5\ \mathrm{cm\ a^{-1}}$ is imposed for $y = \pm 50$ km. The initial temperature is homogeneous except for a weak fluctuation of amplitude 10 K over a width of 10 km. Figure 2 illustrates solutions for two values of the initial background temperature; 600 K and 650 K. In agreement with the comments made above the initially colder case exhibits a much more rapid stage of initiation with rise in temperature and drop in shear stress and viscosity. This stage lasts for about 0.2 Ma compared to slightly over 1 Ma for the warmer case. The velocity profiles are not shown but the shear-strain concentration is more dramatic than in Fig. 1. Indeed the 100 km wide shear zone rapidly narrows to less than 2 km during the stage of initiation. The present cases are very similar to those studied by Yuen *et al.* (1978) except for the fact that the law of deformation is non-Newtonian. Thus, it is interesting to compare the long term behaviour derived from both studies. Here, for

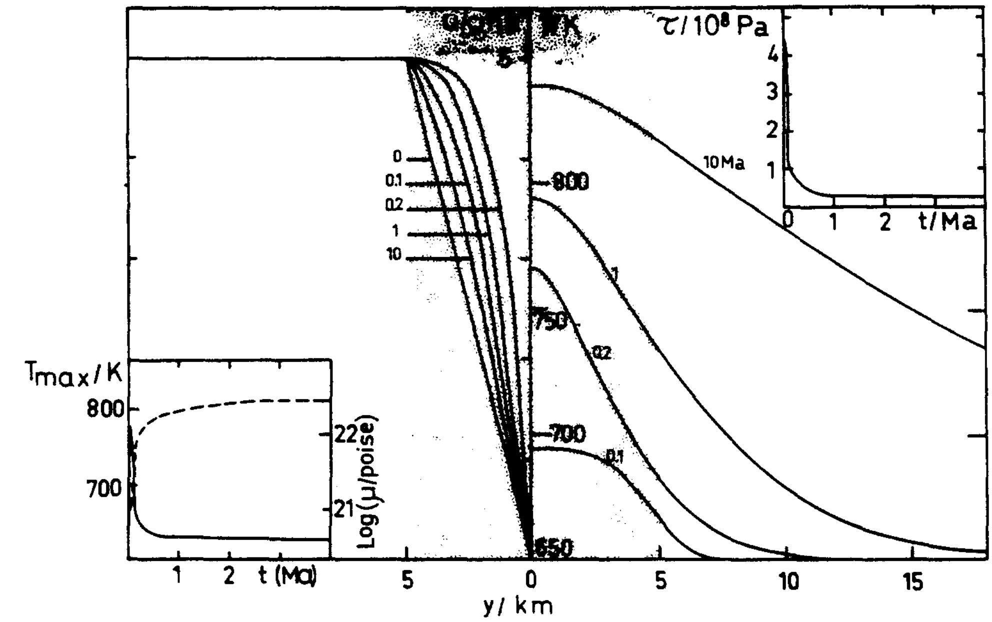

Fig. 1. Temperature and velocity profiles at various times (in Ma) after shearing starts. The shaded area is the zone of weakness bounded by rigid material. Time variations of stress, maximum temperature (dashed line) and minimum effective viscosity are shown in the insets.

times beyond 1 Ma, the viscosity levels off to a value of about 5×10^{20} poise (5×10^{19} Pa s). This is in perfect agreement with the solutions of Yuen *et al.* (1978) and with the analytical formula:

$$\frac{1}{\tau_{xy}^{(n-1)} 2BT_{max}} \exp \frac{Q}{RT_{max}} = \frac{8KR}{u_0^2 Q}. \qquad (6)$$

The value of T_{max} can be inferred from this expression which is obtained by integration of equation (2) assuming steady state. Combined with equation (5) it gives the value of the viscosity minimum μ_{min} in the centre of the shear zone:

$$\mu_{min} = \frac{8KRT_{max}^2}{u_0^2 Q}. \qquad (7)$$

This derivation only applies to materials with a homogeneous law of deformation, that is without intrinsic zones of weakness as in Fig. 1. It is exact for the Newtonian cases and numerically very good for the non-Newtonian cases. As in the solutions of Yuen *et al.* (1978) the width of the thermal anomaly and of the sheared domain are close to being proportional to $\sqrt{\text{time}}$ at large times. Consequently the stress value decreases as $t^{(-1/2)}$. For $n \neq 1$ this decrease has to be

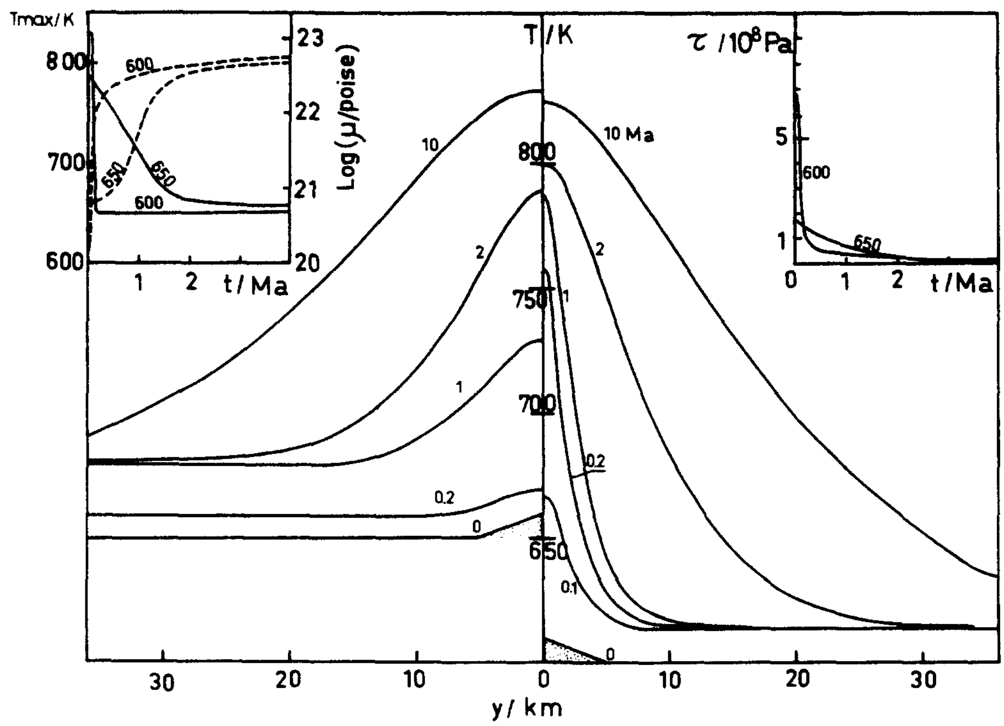

Fig. 2. Temperature profiles at various times for two different initial temperature conditions. The shaded areas mark the initial temperature perturbation. The insets are as in Fig. 1. Indices for the curves refer to initial background temperature.

compensated by a slow increase in the temperature T_{max} as the effective viscosity stays about constant. The temperature drift (see Fig. 2) is however too limited to invalidate the conclusion that melting is unlikely in such situations.

The lithosphere having an isothermal upper boundary (the earth's surface), upward conduction will be present. This will tend to inhibit the widening of the thermal anomaly. Thus two-dimensional models present a steady state width. They are also useful for estimating the surface heat flow.

MELTING BY SHEAR HEATING

Shear of a homogeneous rock mass, with prescribed velocities corresponding to steady plate motion, was shown to generate positive temperature anomalies which soften the material but do not produce melting. Various field observations can be interpreted in terms of syntectonic melting (Bouchez 1977, Nicolas *et al.* 1977). This leads us to investigate further situations. First we consider shear zones of narrow metric or centimetric width which are often found in nature. Then, we study energy storage and release in a viscoelastic system. These considerations are akin to models with constant applied stress, or to the well known stick-slip models which seismologists use for the brittle part of the crust. Finally we show that magmatism in shear zones may simply result from the stratified nature of the lithosphere.

Narrow shear zones

Equation (6) predicts a maximum temperature well below the solidus when shear takes place in a homogeneous medium. We shall therefore examine a case where the deformation is strongly confined because of inhomogeneities in the rock properties. The present model is thus similar to that of Fig. 1, but with a zone of weakness only 20 m wide. The velocity difference u_0 is again 10 cm a^{-1} but the initial temperature has been raised to 800 K. The choice of a colder initial state for a narrow shear zone induces excessively high shear stresses which in reality would lead to brittle failure; or else it would require a softer rheology than that of wet quartzite which is used throughout this paper. Figure 3 shows that the central temperature now rises high enough for melting to take place. This vigorous heating is to be related to the expression of the total heat q produced per unit area over the full width Δl of the sheared zone.

$$q = u_0 \left(\frac{n+1}{n} \right) \left[\frac{T}{2B} \exp \frac{Q}{RT} \frac{1}{\Delta l} \right]^{1/n} . \qquad (8)$$

The smaller the value of Δl the larger the value of q will be. Such a process however implies a rather high total strain, here about 500, before melting occurs. On the other hand, if the velocity difference u_0 is reduced, the time to reach melting becomes geologically irrelevant.

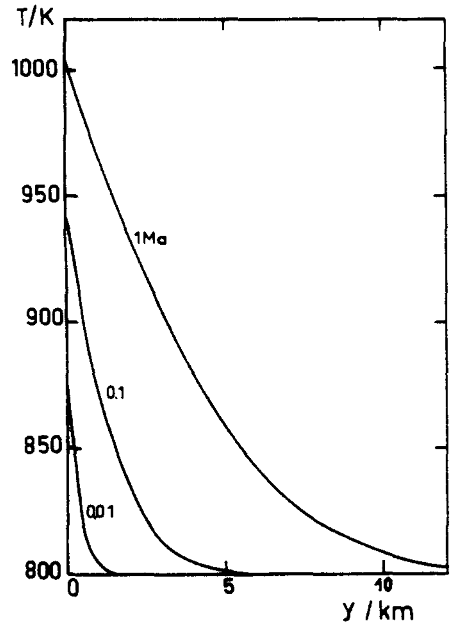

Fig. 3. Temperature profiles at various times for a weakness zone 20 m wide.

Shear zones up to several metres wide are commonly observed (Boissière & Vauchez 1978, Burg & Laurent 1978). If the corresponding displacements have velocities not exceeding centimetres per year, no detailed computation is necessary to demonstrate that these cannot have generated sizable thermal anomalies. Indeed, the time required to produce a given temperature increase ΔT for a constant heat production ($\tau_{xy}\, u_0$) by unit area on the fault would be the following: $t = \dfrac{\pi\rho C^p K \Delta T^2}{\tau_{xy}^2 u_0^{\,2}}$ (Carslaw & Jaeger 1959). Correspondingly the total displacement would be $u_0 t = \dfrac{\pi\rho C_p K \Delta T^2}{\tau_{xy}^{\,2} u_0}$. Thus the higher u_0 and τ_{xy}, the shorter the duration and the displacement. To achieve a temperature increase ΔT, of say 100 K, assuming τ_{xy} as large as 5 kbar (0.5 GPa) and u_0 = 10 cm a^{-1} one finds a minimum time of about 3000 a and a minimum displacement of about 300 m. If u_0 = 1 mm a^{-1} is felt to be more realistic, those figures become 30 Ma and 30 km. Observed strains seem to be too small, say 20, to be compatible with such displacements across centimetric shear zones. In reality either ΔT has to be negligible or high velocities have to take place over short duration (creep event). Our opinion is that small scale shear zones imply a softening mechanism other than shear heating.

Shear relaxation in a viscoelastic material

One may wonder if stress accumulation can produce shear instabilities in a ductile medium. This is the only possibility for reaching displacement velocities of larger magnitude than those of steady plate motion, if no body force is applied. Following Griggs & Baker (1969), let us consider a zone of weakness of finite width w, that is a situation similar to that described in Fig. 1, but with the possibility of accumulating elastic energy in a broader domain of width d = 40 km. The choice of this value is not easy. In reality d is of the order of the length of the fault segment along which stress can be accumulated. The computation procedure in this case is equally based on equations (1), (2) and (3) but the stress value derives from the following equation:

$$\dot{\tau}_{xy} = \frac{E}{d} \int_0^t (u_0 - u_w)\, dt. \quad (9)$$

Here t is the time since stress accumulation started, u_w the velocity across the weakness zone related to viscous strain and E the Youngs modulus taken to be equal to 50 GPa. What happens if, starting from an unstressed state, one imposes a velocity difference u_0 across the width d? If the zone of weakness is as broad as in Fig. 1, the thermal anomaly caused by shear heating behaves equally gently: a steady state velocity u_0 is achieved across the ductile region. For a narrower zone of weakness (w = 100 m) Fig. 4 shows that the release of accumulated stress has been able to generate a narrow peak in temperature in a time span of about 100 a. This was preceded by a period of stress build up and slow heating lasting 5000 a during which the temperature has risen by several tens of degrees above the initial value of 700 K. The total displacement across the ductile region amounts to 100 m over the last 100 a, including 20 m over the last couple of years. Thus different widths yield contrasting behaviour. It proves that the available stored energy has to be dissipated in a relatively narrow region to lead to a thermal and mechanical runaway. During the last 100 a the shear stress remains fairly high dropping from 4.8 to 3 kbar (0.48–0.3 GPa). Then the melting temperature is reached so that this fast creep event could be followed by a seismic displacement. The large magnitude of these preseismic and seismic displacements present in all solutions leading to thermal runaway call for serious objections when compared with the geophysical record. Therefore accumulation of high stresses over extended volumes followed by rapid relaxation is unlikely. The conclusion to be drawn from the above one-dimensional model is the following: slow stress accumulation only leads to fast creep events of moderate amplitude, and therefore not to melting. One can of

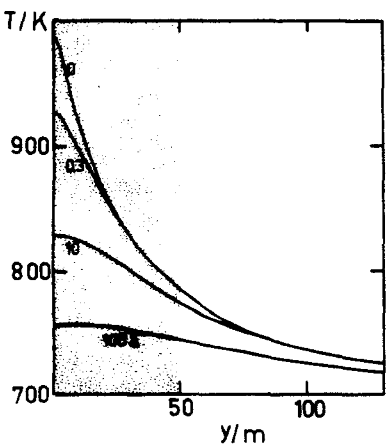

Fig. 4. Thermal runaway. Temperature profiles for various times before melting starts in the centre (y = 0). The shaded area is the zone of weakness bounded by purely elastic material. The elastic region is 40 km wide.

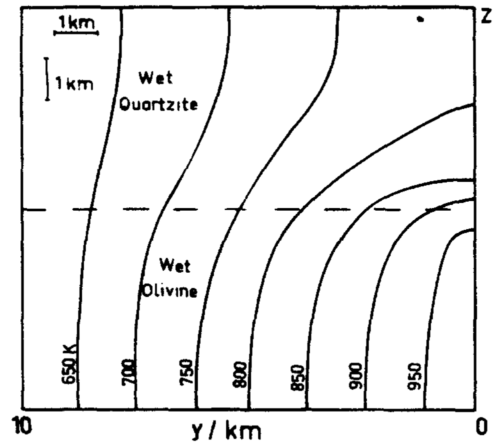

Fig. 5. Isotherms in a two layer block sheared normal to the drawing. Only half of the block is shown. The dashed line is the interface between the two rock types. The half width of the sheared area is about 2 km.

course speculate that in three-dimensional situations melting could still occur after relatively short preseismic displacements, if high stresses have been rapidly built-up in a confined region.

Shear and melting in a stratified medium

The Earth's lithosphere is made up of a variety of rock types with contrasting mechanical properties. The simplest geometry is that of a stratified system. If the plane of shear cuts across the layers a new interesting situation arises. Shear heating induces higher temperatures in a layer made of hard material than in a layer made of soft material. On the other hand the melting point of a soft rock is usually lower than that of a hard rock. Melting of the softer layer can thus take place because of indirect heating through the interface. A quantitative test of this phenomenon is shown in Fig. 5. Here the soft rock is wet quartzite and the hard one is wet olivine for which we use the law of deformation (1) with $B = 1.\ 1 \times 10^{-10}$ks^{-1} Pa^{-3}, $n = 3$, and $Q = 95$ kcal mol^{-1} (4×10^5 J mol^{-1}). The central shear plane ($y = 0$) is perpendicular to the interface of the two layers. A temperature of 600 K is imposed on both sides at $y = \pm$ 10 km. The imposed temperature profile at the bottom and top of these blocks corresponds to steady state far away from the interface. The imposed velocity is perpendicular to the plane of the figure and differs by a quantity u_0 = 10 cm a^{-1}, between y = + 10 km and y = −10 km. The bending of the computed isotherms illustrates the heat transfer across the interface. Temperatures in excess of 900 K capable of producing melting are found in the quartzite layer within the shear zone near the interface. The same geometry can be found between basement rock and sediments: hence the possible anatexis of sedimentary formations in shear zones.

The above solution is derived from the integration of equations (1) and (3), combined with a steady state temperature equation including a term for conduction along the z direction. The velocity profile varies with z. This implies the existence of shear stresses τ_{xz} in planes parallel to the interface. Their minor contribution to shear heating has been neglected in the calculation.

The particular simple geometry used above just helps to emphasize the fact that heterogeneities can strongly influence the conclusions concerning melting.

CONCLUSION

High temperature creep experiments on minerals which are carried out close to the melting point yield effective viscosities of the order of 10^{15} poises (10^{14} Pa s). For rocks, the same statement holds: in granite for example, partial fusion occurs when the effective viscosity drops to the above value. Extrapolation of these results to half the melting temperature indicates increases in viscosity by ten orders of magnitude. The mechanical behaviour of the lithosphere reflects an average viscosity of at least 10^{25} poises (10^{24} Pa s). On the other hand, geophysical estimations for the upper mantle give values of 10^{21} poises (10^{20} Pa s). In the model shear zones analysed in the first part of this paper, shear heating is so efficient that it leads to local lithospheric viscosity values comparable to those of the asthenosphere. For lower values the heat produced by mechanical dissipation would be too small to compensate the heat loss by conduction. For example, with a velocity difference of 10 cm a^{-1} across a 1 km wide zone of viscosity 10^{15} poises (10^{14} Pa s) the dissipated energy amounts to much less than that of crustal radioactivity.

Our models account for both an initial phase of rapid thermal weakening and shear concentration, and a long term regime with stabilised viscosity and progressive widening of the structure. The sheared region is much narrower than the induced thermal anomaly. The effect of the non-Newtonian rheology is only that a weak positive temperature drift accompanies the stress decrease during the second phase. It should be emphasized that the efficiency of shear heating is limited to situations where a relatively hard rock mass is deformed with a large enough differential velocity, say 1 cm a^{-1} or more. Indeed equation (7) indicates that the induced viscosity minimum, μ_{min}, is inversely proportional to the square of the differential velocity u_0. For $u_0 = 10$ cm a^{-1} the temperature rises up to the point where μ_{min} becomes as small as about 5.10^{20} poises. For $u_0 = 1$ mm a^{-1}, the effect may almost vanish, the value of μ_{min} being 5.10^{24} poises.

This paper reinforces the conclusion that melting induced by shear heating is something very unlikely in most circumstances. However, we have introduced particular geometries and boundary conditions in order to investigate further this aspect. One might even argue that shear concentration is a consequence of partial melting in rocks, not a cause. The other extreme view is to invoke time independent stresses which, if large enough, certainly would lead to melting. It seems to us that energy storage and release due to viscoelastic behaviour of rocks is more appropriate. We found this may lead to a variety of solutions from well behaved shear processes to catastrophic runaways, without forgetting creep events of reasonable amplitude. As far as one-dimensional models can be trusted, thermal runaways require unrealistic displacements.

Of the two geometries shown in the second part of our paper (narrow weakness zones and shearing across stratified layers), the second one seems to bear a wider potential for magma production. It relies on the contrasting amplitude of shear heating anomalies in different types of rocks.

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REFERENCES

Bossière, G. & Vauchez, A. 1978. Déformation naturelle par cisaillement ductile d'un granite de Grande Kabylie occidentale (Algérie). *Tectonophysics* **51**, 57–81.

Bouchez, J. L. 1977. Le Quartz et la Cinématique des Zones Ductiles. Unpublished thèse, Université de Nantes.

Brun, J. P. & Cobbold, P. 1980. Strain heating and thermal softening in continental shear zones: a review. *J. Struct. Geol.* **2**, 149–158.

Burg, J. P. & Laurent, Ph. 1978. Strain analysis of a shear zone in a grandiorite. *Tectonophysics* **47**, 15–42.

Carslaw, H. S. & Jaeger, J. C. 1959. *Conduction of Heat in Solids.* Oxford University Press, Oxford.

Cooper, R. F., Kohlstedt, D. L., Weathers, M. S. & Bird, J. M. 1978. Dislocation structure in deformed quartz grains from the Moine thrust fault, Scotland. *Eos.* **59**, 1207.

Griggs, D. T. & Baker, D. W. 1969. The origin of deep-focus earthquakes. In: *Properties of Matter under Unusual Conditions* (edited by Mark, H. and Fernback, S.) Interscience, New York, 23–42.

Nicolas, A., Bouchez, J. L., Blaise, J. & Poirier, J. P 1977. Geological aspects of deformation in continental shear zones. *Tectonophysics* **42**, 55–73.

Parrish, D. K., Krivz, A. L. & Carter, N. L. 1976. Finite element folds of similar geometry. *Tectonophysics*, **32**, 183–207.

Poirier, J. P. 1980. Shear localization and shear instability in materials in the ductile field. *J. Struct. Geol.* **2**, 135–142.

Scholz, C. H. & Hanks, T. C. 1978. Metamorphism, argon depletion, heat flow and stress on the Alpine fault. *Eos.* **59**, 1206.

Turcotte, D. L. & Oxburgh, E. R. 1968. A fluid theory for the deep structure of dip-slip fault zones. *Phys. Earth Planet. Interiors* **1**, 381–386.

Yuen, D. A., Fleitout, L., Schubert, G. & Froidevaux, C. 1978. Shear deformation zones along major transform faults and subducting slabs. *Geophys. J. R. astr. Soc.* **54**, 93–120.